Crack localization in stepped rotors based on Bayesian fusion of multi-scale super-harmonic characteristic deflection shapes

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Abstract

A crack is one of the most dangerous faults in rotors of key rotating machinery. In order to make reasonable maintenance, crack localization for rotors under operational conditions is very important and full of challenges. Focusing on crack localization in rotating rotors, a new crack localization method for stepped rotors based on Bayesian fusion of multi-scale super-harmonic characteristic deflection shapes (SCDSs) is proposed. The nonlinearity induced by cracks is utilized to eliminate the interference of stiffness reduction by steps in the rotor, and the noise-robustness issue is tackled by casting the SCDSs into multi-scale SCDSs with multi-scale space theory. Moreover, Bayesian fusion is applied to the multi-scale SCDSs to derive a new damage index to localize cracks. Numerical and experimental investigations are conducted to validate the proposed method. The results indicate that the proposed method is effective, accurate and robust for single or multiple crack localization in stepped rotors, and thus it has a great potential in practical applications.

Keywords: Crack localization; rotors; Bayesian fusion; nonlinear; super-harmonics; characteristic deflection shape.

1. Introduction

Cracks in rotors of rotating machines may lead to catastrophic failures, if not detected and dealt with timely. In view of the importance, many investigations have been performed for crack monitoring in rotors, which makes up a most important direction in prognostics and health management (PHM) [1] or system/structural health monitoring (SHM) [2]. Among these investigations, vibration-based methods are widely investigated and recognized as useful tools. PHM for cracked rotors can be categorized into four levels [3] as crack detection [4-10], crack localization [11-13], crack identification [14-17] and remaining useful life prediction [18, 19]. These four stages can provide information required when making a reasonable maintenance strategy, which is of great significance to guarantee the safe operation and reduce maintenance cost of key rotating

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machines. Among the four stages, most research efforts have been focused on crack detection, such as signal-features-based methods and machine-learning- based methods, thus crack detection is relatively mature. However, for crack localization in rotating rotors, there are still big challenges. How to avoid the requirement of prior information such as accurate models of rotors and how to reduce the interference of noise and other typical factors and develop more robust and practical methods, are of great importance for crack localization.

For crack localization, spatial information is required. One of the most widely adopted methods is based on mode shapes or their spatial derivatives. Because of the difficulties in measuring mode shapes for operating rotors, these kinds of methods are commonly applied for crack localization in stationary structures. Hu et al. [20] proposed a damage localization method based on Bayesian operational modes. Altunışık et al. [21] realized crack localization of a beam from its mode curvature. Katunin [22] adopted the wavelet transform of mode shapes to localize cracks. Guo et al. [23] validated the effectiveness of crack localization for beam and plate structures from mode shapes through numerical and experimental investigation. As mode shapes are normally extracted according to a theory based on linear assumptions, they cannot well reflect the nonlinearity of a breathing fatigue crack. For example, a mode shape-based method does not involve the nonlinearity information which will lead to the missing of some important unique features. Moreover, for operating rotors, mode shapes are relatively difficult to extract and the mode curvature-based methods will amplify and propagate the effects of noise when calculating the curvature by a finite difference scheme.

Inspired by the localization methods based on mode shapes, methods based on characteristic deflection shapes (CDSs) by Cao and Ouyang [24] are investigated for crack localization in stationery structures and rotors. The CDS is a certain spatial shape extracted from responses at multiple measurement points following a certain rule, therefore, it includes but not limited to a mode shape or an operational deflection shape (ODS) [25]. As one kind of CDSs, the kurtosis of ODS was utilized by Saravanan et al. [26] to localize cracks in operating rotors. Babu et al. [27] proposed the amplitude deviation curve derived from ODSs to localize cracks in a rotating rotor. Singh et al. [28] experimental investigated crack localization in rotors by detecting the crack-induced discontinuities in shaft deflections at regular axial locations of the shaft and with different excitation frequencies. That method was proved effective to realize multi-crack localization in a stepped shaft with an intact reference model. However, the reference model of an intact system is rather difficult to obtain, if there is no baseline, numerous methods will not work.

The transmissibility of operational deflection shape (TODS) was defined by Li et al. [29] for crack localization in beam-like structures by using the higher order harmonic ODS. A residual ODS-based method considering higher harmonic components of exciting frequency was developed by Asnaashari et al. [30] to localize cracks in a rotor, by which the effects of the fundamental frequency excitation can be eliminated. Prawin et al. [31] utilized the crack-induced nonlinear components and proposed an approach based on the spatial curvature of Fourier power spectrum from multiple measurement points as a damage-sensitive feature for breathing crack localization. Broda et al. [32] exploited the first two super-harmonics for localization of a breathing crack by evaluation of the ratio of a super-harmonic to the fundamental harmonic at various spatial locations. In addition, a detailed literature review of breathing crack identification techniques for structures was presented by Bovsunovsky and Surace [33]. These studies indicated that the CDS derived from higher order frequency components seemed more sensitive to nonlinear breathing cracks.

As well known, crack-induced local stiffness reduction will introduce discontinuities or distortions in the elastic line of the shaft. In addition, steps in rotating machines are always existed and will also cause discontinuities or distortions at their locations. Consequently, steps will interfere with the crack localization. If no baseline model and prior information about the steps are available, the results of crack localization could be misguided. However, cracks in rotors are normally fatigue ones and they will open and close with the rotation of rotors which is called breathing and will make the system nonlinear. Thus, crack breathing phenomenon is a main distinction between fatigue cracks and steps. Hence, cracks could be determined exclusively, if nonlinearity and stiffness reduction caused by the cracks were utilized simultaneously, regardless of the steps in a rotor. In addition, the crack induced features are normally quite weak, which are readily submerged by the measurement noise or other uncertainties. Therefore, how to tackle the noise-robustness issue of the crack localization method is another motivation of this work.

Considering the distinction between nonlinear cracks and linear steps in the super-harmonic characteristics, a new crack localization method based on Bayesian fusion of multi-scale super-harmonic characteristic deflection shapes (SCDSs) is proposed to realize crack localization with the interference of steps for operating rotors. The basic principle is to detect the crack induced local shape distortions in the SCDSs, which are extracted by frequency domain decomposition at super-harmonic frequencies. To tackle with the noise problem and enhance the robustness of the method, the SCDSs are transformed into multi-scale SCDSs by Gaussian multi-scale analysis, and the multi-scale SCDSs are used to derive a new damage index based on Bayesian fusion theory. Numerical simulations for rotors with steps and breathing cracks are carried out by the finite element method and linear fracture mechanics with crack closure line breathing model based on strain energy release rate approach, and the numerical results indicate that the proposed method is effective, accurate and robust, which is also validated by experimental results. Moreover, the crack localization results by the proposed method are found to outperform those by a recently published method.

The major original contributions of the present work are as follows:

1. A new crack localization method based on Bayesian fusion of multi-scale SCDSs for operating rotors with single or multiple cracks under the interference of steps has been proposed and validated by numerical and experimental studies.
2. The proposed method is robust to strong noise owing to multi-scale information fusion, which could overcome the limitation of the majority of crack localization methods.
3. The proposed method is output-only and baseline-free which will be quite practical for real machines. It should be mentioned that the excitation and baseline information of an operating rotor is quite difficult to obtain due to environmental and operational constraints.

The structure of the paper is organized as follows. In Section 2, the proposed crack localization method is presented in detail. In Section 3, numerical simulations are investigated for rotors with different breathing crack and step configurations. After this, the proposed method is validated by experimental research in Section 4. In order to show the priority of the proposed method, a comparison between the proposed method and a former published crack localization method is made in section 5. Finally, some key conclusions are drawn in Section 6.

2. Descriptions of the proposed crack localization method

2.1 Concept of super-harmonic characteristic deflection shapes

The concept of 'super-harmonic characteristic deflection shapes' was defined in a previous work [34] of the authors', but for the convenience of understanding this current paper, the concept will be briefly introduced first. Let $\mathbf{y}_i$ be the response from sensor *i*, then the system response matrix $\mathbf{Y}$ can be constructed by the simultaneously measured responses from the *n* distributed sensors along the shaft:

$$\mathbf{Y} = [\mathbf{y}_1, \dots, \mathbf{y}_n] = \begin{bmatrix} y_{11} & \cdots & y_{1n} \\ \cdots & \cdots & \cdots \\ y_{m1} & \cdots & y_{mn} \end{bmatrix} \tag{1}$$

where *m* is the length of each response.

Then, the power spectral density matrix $\mathbf{G}_{yy}$ can be expressed as the Fourier transform of the correlation matrix $\mathbf{R}_{yy}$:

$$\mathbf{G}_{yy}(\omega) = \mathcal{F}\left[\mathbf{R}_{yy}(\tau)\right] \tag{2}$$

And the correlation matrix can be obtained by:

$$\mathbf{R}_{yy}(\tau) = \begin{bmatrix} \mathbf{R}_{11} & \cdots & \mathbf{R}_{1n} \\ \vdots & \mathbf{R}_{ij} & \vdots \\ \mathbf{R}_{n1} & \cdots & \mathbf{R}_{nn} \end{bmatrix} \tag{3}$$

where $\mathbf{R}_{ij}$ is the correlation function between $\mathbf{y}_i$ and $\mathbf{y}_j$ and E is the averaging operator, and $\mathbf{R}_{ij}$ can be expressed as:

$$\mathbf{R}_{ij}(\tau) = E\left[\mathbf{y}_i(t)\mathbf{y}_j(t+\tau)\right] \tag{4}$$

Therefore, the power spectral density matrix at frequency ω_i can be decomposed by singular value decomposition as:

$$\mathbf{G}_{yy}(\omega_i) = \mathbf{U}_i\mathbf{S}_i\mathbf{V}_i^{\mathrm{T}} \tag{5}$$

where $(\boldsymbol{U}_i)_{n\times n} = [\boldsymbol{u}_{i1}, \dots, \boldsymbol{u}_{in}]$ and $(\boldsymbol{V}_i)_{n\times n} = [\boldsymbol{v}_{i1}, \dots, \boldsymbol{v}_{in}]$ are orthogonal matrices containing the left and right singular vectors $(\boldsymbol{u}_{ij})$ and $(\boldsymbol{v}_{ij})$ respectively; $(\mathbf{S}_i)_{n\times n}$ is a diagonal matrix with singular values at the diagonal entries.

Because the power spectral density matrix is a square and positive definite matrix, the eq. (5) becomes:

$$\mathbf{G}_{yy}(\omega_i) = \mathbf{U}_i\mathbf{S}_i\mathbf{U}_i^{\mathrm{T}} \tag{6}$$

According to singular value decomposition, the first singular vector $\boldsymbol{u}_{i1}$ contributes the most to the vibration of a structure at frequency ω_i, so it can be taken as the dominant feature vector of $\mathbf{G}_{yy}(\omega_i)$ [35]. Therefore, the first singular vector $\boldsymbol{u}_{i1}$ can be considered a characteristic property of all the measured responses at frequency ω_i and its shape is defined as a characteristic deflection shape (CDS). Further, if the frequency is a super-harmonic of the fundamental frequency, then the corresponding CDS is called a super-harmonic CDS (SCDS).

For the rotating machinery, the fundamental frequency is the rotating frequency. When there are cracks in the rotor, super-harmonic components will be induced by the asymmetry and breathing of cracks. Therefore, if the SCDSs at their super-harmonic frequencies can be extracted for crack localization, other interference from linear features such as steps could be eliminated as no nonlinear super-harmonics will be generated by them.

2.2 New Damage indexes from Bayesian fusion of multi-scale super-harmonic characteristic deflection shapes

With SCDS obtained by frequency domain decomposition, the crack localization can be accomplished by amplifying the distortions via evaluating the curvature of SCDS or the after-treatment methods such as gapped smoothing method (GSM), which is proposed for crack localization in rotating rotors with steps in the authors' previous work [34]. However, robustness of this method is still an unsolved issue to be addressed. The main idea to improve the robustness of the method is to transfer the SCDS into multi-scale SCDS by using multi-scale space analysis, then to combine the multi-scale SCDS with the Teager energy operator and Bayesian fusion methods to derive a new damage index to reduce the effect of measurement noise. The detailed procedures are presented in the following.

2.2.1 Multi-scale super-harmonic characteristic deflection shapes by Gaussian multi-scale analysis

Though crack localization based on SCDS and GSM can reflect local cracks accurately without baseline model of the healthy rotor, these methods are not robust against noise, especially when the extracted SCDSs are contaminated by strong noise which is common for rotating rotors. In order to deal with this defect, the multi-scale space theory is applied for the extracted SCDS to reduce the noise in multi-scale space. The multi-scale SCDS is expressed as:

$$L(x,\sigma) = G(x,\sigma)\otimes\phi(x) \tag{7}$$

where x is the position variable corresponding to measurement point in this work, σ is the scale parameters, $\otimes$ is the convolution operator, ϕ is the extracted SCDS and G is the Gaussian kernel, which is defined as:

$$G(x,\sigma) = \frac{1}{2\pi\sigma^2}e^{-\frac{x^2}{2\sigma^2}} \tag{8}$$

2.2.2 Bayesian fusion of multi-scale super-harmonic characteristic deflection shapes

After obtaining the multi-scale SCDS by Gaussian multi-scale analysis (GMA), to enhance the weak singularities induced by cracks in all the scales of the SCDS, the Teager energy operator (TEO) is adopted to get the multi-scale TSCDS, which is the TEO of the multi-scale SCDS and is defined as:

$$T(x,\sigma) = L^2(x,\sigma) - L(x-1,\sigma)L(x+1,\sigma). \tag{9}$$

In the following, a new robust damage index is proposed by using the Bayesian fusion (BF) of the multi-scale TSCDS. From BF theory, the multi-scale TSCDS with any scale parameter is viewed as an information source. Now, M scales are used for information fusion, and the corresponding multi-scale TSCDSs are denoted as information sources S_1 to S_M. The crack located at measurement point i is view as an object A_i, therefore, there will be NE objects considered corresponding to the measurement point number. Moreover each object is assumed to be mutually exclusive and obeys the uniform distribution, as crack information is unknown a priori, and the crack can present at any position in the rotor. Therefore, the damage probability of each object in one scale or the prior probability can be defined as:

$$P(\mathrm{A}_i) = \frac{1}{NE} \tag{10}$$

Then, the probability that there is a crack at measurement point *i* under all scales can be expressed by Bayesian fusion of all the *M* information sources as:

$$P(\mathrm{A}_i|S_1, S_2, \dots, S_M) = \frac{P(\mathrm{A}_i)\prod_{k=1}^{M} P(S_k|A_i)}{\sum_{j=1}^{NE} P(\mathrm{A}_i)\prod_{k=1}^{M} P(S_k|A_i)}, \tag{11}$$

where $P(\mathrm{A}_i|S_1, S_2, \dots, S_M)$ is taken as the damage index (DI) which is called 'improved DI' in this work, the larger the value, the higher probability there is a crack at corresponding location. Moreover, the total probability $\sum_{i=1}^{M} P(\mathrm{A}_i|S_1, S_2, \dots, S_M)$ equals to 1, which indicates there is an assumption that at least one crack is present in the rotor. Therefore, the proposed method is effective just for the cases when there are cracks in rotors. Thus, crack detection is necessary to guarantee the existing of cracks before the application of the proposed crack localization method. This is one limitation of the proposed method, but, fortunately, crack detection is relatively mature and reliable. And $P(\mathrm{S_k}|\mathrm{A_i})$ denotes the probability that there is a crack at measurement point *i* under the *k*th scale, which can be expressed as:

$$P(\mathrm{S_k}|\mathrm{A_i}) = \frac{T(x_i, \sigma_k)}{\sum_{j=1}^{NE} T(x_j, \sigma_k)}. \tag{12}$$

where, the larger the value of $T(x_i, \sigma_k)$**,** the higher probability there exists a crack at the place of x_i under the *k*th scale.

3. Numerical simulations

3.1 Modelling of a cracked and stepped rotor system

To investigate the proposed crack localization method, numerical simulations are performed for a rotor-bearing system with steps, which is depicted in Fig. 1. Two-node Timoshenko beam elements are utilized to discretize the rotor and three translational and three rotational degrees-of-freedom (DOFs) per each node are considered. The discs are simplified as rigid bodies containing lumped inertias in the six DOFs at the corresponding nodes, and the gyroscopic effect is also taken into account. The two bearings are assumed as isotropic spring-damping systems. The torsional and axial DOFs of the rotor in the driving end are fixed. The cracked element is shown in Fig.1 (b), and its stiffness matrix is derived from linear fracture mechanics with Crack Closure Line Position (CCLP) method [36]. By assembling the matrices of the cracked elements and un-cracked elements, the motion equation of the rotor in the stationary coordinate system can be constructed as:

$$\mathbf{M}\ddot{\mathbf{q}} + \left(\mathbf{D} + \Omega\mathbf{D}_\mathrm{g}\right)\dot{\mathbf{q}} + \mathbf{K}(t)\mathbf{q} = \mathbf{F}_\mathrm{u} + \mathbf{F}_\mathrm{g} + \mathbf{F}_\mathrm{ex} \tag{13}$$

where $\mathbf{M}$ is the system mass matrix, $\mathbf{D} = a\mathbf{M} + b\mathbf{K}$ is the system Rayleigh damping matrix, Ω is the rotating frequency, $\mathbf{D}_\mathrm{g}$ is the system gyroscopic matrix, $\mathbf{K}(t)$ is the system stiffness matrix which is time varying with the crack breathing, $\mathbf{F}_\mathrm{u}$ is the unbalance excitation force vector, $\mathbf{F}_\mathrm{g}$ is the gravitational excitation force vector, $\mathbf{F}_\mathrm{ex}$ is the external excitation force vector, $\mathbf{q}$ is the displacement vector of *n* nodes which can be expressed as:

$$\mathbf{q} = \{\mathbf{q}_1, \mathbf{q}_2, \dots \mathbf{q}_i, \dots, \mathbf{q}_n\}^{\mathrm{T}} \tag{14}$$

here $\mathbf{q}_i$ is the displacement vector of node *i*:

$$\mathbf{q}_i = \{x_i, y_i, z_i, \theta_{xi}, \theta_{yi}, \theta_{zi}\}^{\mathrm{T}} \tag{15}$$

The detailed expressions of the variables were presented in the authors' previous paper [34].

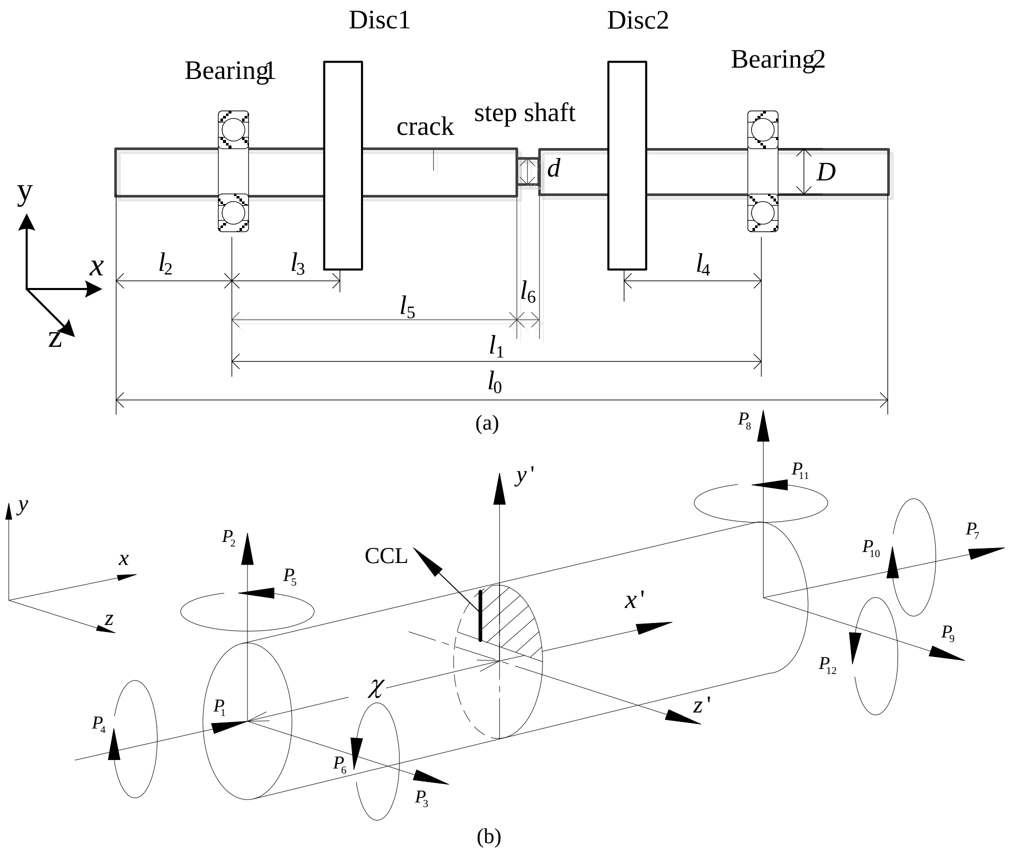

Fig. 1 (a) Schematic diagram of a cracked rotor-bearing system with steps; (b) Model of a crack element

The detailed parameter values of the rotor in Fig.1 are presented in Tab. 1. The rotor is discretized into 60 equivalent beam elements, and cracks with different parameters are embedded using the cracked shaft elements. All the cracks considered are transverse breathing ones for simplicity (slant cracks can be also modelled by the modelling method in this work, which will hardly affect the feasibility of the crack localization method) and no propagating of the cracks is assumed during the measurement of responses. The finite element model of the rotor is depicted in Fig. 2. The first natural bending frequency of the rotor without crack and step is 42.1 Hz, which is experimentally verified by a modal experiment [15]. As the crack breathes with time and its breathing behaviour is dependent on the response, which makes the stiffness of the system time-varying and state-dependent nonlinear. Therefore, the Newmark method [37] is utilized to obtain the time-domain responses. The sampling frequency is set as 5000Hz to capture breathing of cracks. Rotating speed is set at 840r/min, which is 1/3 of the first critical rotating speed, to make the 2X frequency component away from the critical rotating frequency and observable amplitude.

Tab. 1 Parameters of the stepped rotor-bearing system

Parameter	**Value(units)**
Shaft length (l_0)	0.6m
l_1	0.46m
l_2	0.07m
l_3	0.08m
l_4	0.1m
l_5	0.26m
l_6	0.01m
Shaft diameter (D)	0.01m
Stepped shaft diameter(d)	0.008m
Density (40Cr)	7.87×10^{3}kg.m^{-3}
Young’s modulus	2.11×10^{11}Pa
Poisson’s ratio	0.277
Mass of disc 1	0.759 kg
Polar moment of inertia of disc 1	5.758×10^{-4} kg.m^{2}
Diametrical moment of inertia of disc 1	3.177×10^{-4} kg.m^{2}
Mass of disc 2	0.770 kg
Polar moment of inertia of disc 2	5.843×10^{-4} kg.m^{2}
Diametrical moment of inertia of disc 2	3.232×10^{-4} kg.m^{2}
Gravitational acceleration	9.8m.s^{-2}
Rayleigh damping coefficient (a)	0.68
Rayleigh damping coefficient (b)	2.8×10^{-5}
Bearing stiffness coefficient (k_b)	9.6×10^{-5}N.m^{-1}
Bearing damping coefficient (c_b)	100N.s.m^{-1}

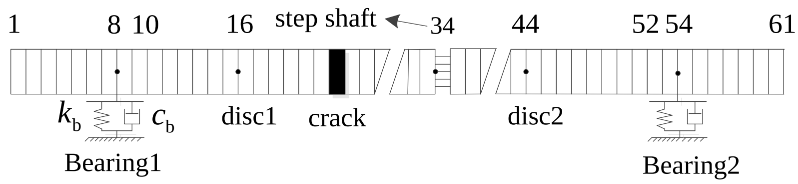

Fig.2 The finite element model of the rotor-bearing system

Measurement points are arranged at each node from node 10 to node 52, which almost covers the shaft between the two bearings. In order to verify the proposed method, the rotors with breathing cracks and with or without steps are considered in the numerical simulations, in which the crack depth is set as 15% of the shaft diameter. The crack and step configurations are listed in Tab. 2.

Tab. 2 Configurations of cracks and steps in numerical simulations

Simulation number	Crack 1 (measurement point)	Crack 2 (measurement point)	Position of the stepped shaft (measurement point)
1	21-22	--	--
2	--	31-32	25-26
3	17-18	31-32	25-26

3.2 Localization results based on Bayesian fusion of multi-scale super-harmonic characteristic deflection shapes

In order to verify the feasibility and effectiveness of the proposed method, the three simulations in

Tab.2 are used to perform crack localization. The localization results of the three simulations using the first three SCDSs under no noise condition are shown in Figs.3-5. In each figure, the top graph shows the extracted SCDSs, and the bottom graph is the corresponding localization result which is depicted by the variability of Improved DI with measurement point.

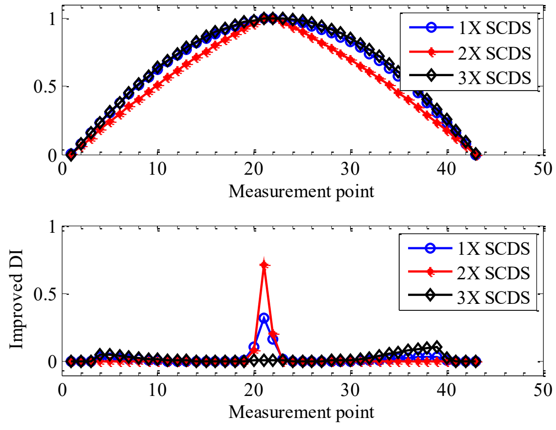

Fig. 3 Localization results for the rotor in simulation 1 without noise from the first three SCDSs

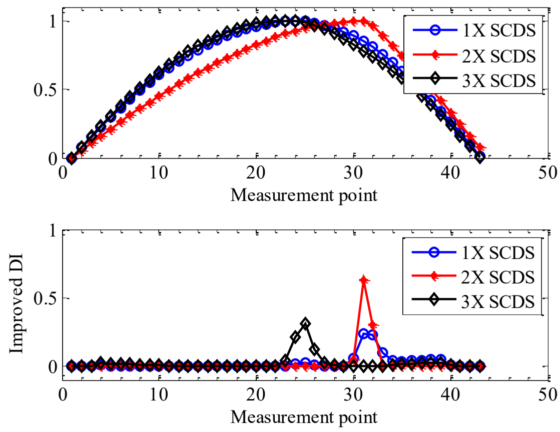

Fig. 4 Localization results for the rotor in simulation 2 without noise from the first three SCDSs

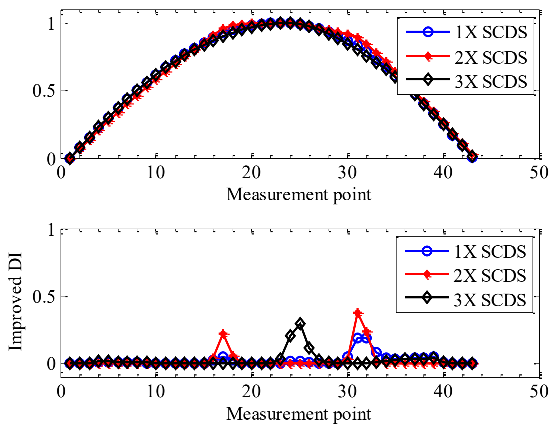

Fig. 5 Localization results for the rotor in simulation 3 without noise from the first three SCDSs

Figs.3-5 demonstrate that the 2X SCDS is efficient for crack localization, where peaks of DI are only present at the positions of actual cracks, thus indicate that the proposed method is effective in localizing the cracks while circumventing the interference of the steps. However, the 1X and 3X SCDSs cannot avoid the interference of steps, and peaks appear at the positions of the steps, which could be misleading, if there is no prior information about the steps and the number of cracks. Whether the selected SCDS contains the interference of steps depends on whether it is dominated by certain linear mode or not. If it is dominated by a linear mode, then the interference of steps cannot be eliminated. As for the 1X SCDS, it is dominated by the first linear mode excited by the synchronous driving frequency, no matter what the rotating frequency is. Therefore, the 1X SCDS cannot eliminate the interference of the local stiffness reduction induced by steps. While, for the 3X SCDS, because the selected rotating speed of the rotor is 840r/min, which is almost equal to the 1/3 critical speed of the rotor, thus the 3X frequency is close to the first natural frequency, so the first linear mode is dominant. Therefore, the 3X SCDS also cannot avoid the interference of steps at this speed. This difficulty can be overcome by decreasing the selected rotating speed and shifting the frequency of 3X component away from the first critical speed, which was validated to be effective in [34], and will not be discussed in the present work. Therefore, only the 2X SCDS is chosen in the following investigation.

In the above investigation, no noise is considered. However, the measured SCDSs are commonly polluted by noise in practice. In order to evaluate the robustness of the proposed method, crack localization for the rotor in simulation 3 with different noise levels is studied. The white noise-polluted SCDS $\mathbf{y}_\mathrm{N}$ can be obtained by:

$$\mathbf{y}_\mathrm{N} = \mathbf{y} + \frac{L_\mathrm{N}\sqrt{\sum(y_i-\mu)^2}}{N}\mathbf{r} \tag{16}$$

where $\mathbf{y}$ is the unpolluted SCDS with length N. L_N is a constant noise level ranging from 0 to 1. μ is the mean value of $\mathbf{y}$. $\mathbf{r}$ is the random number vector in standard normal distribution.

The crack localization results of the proposed method under different noise levels are given in Fig.6.

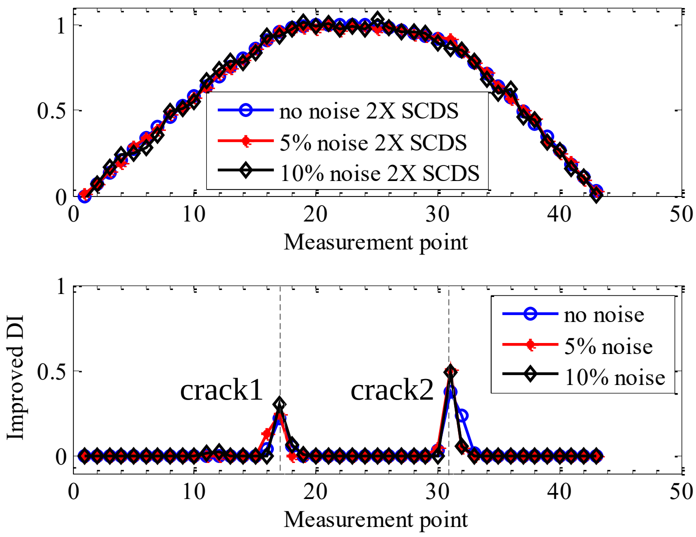

Fig. 6 Localization results for the stepped rotor with two cracks under different level of noise from the 2X SCDS

As one can see from Fig.6, two clear peaks are present at the locations of the cracks, but there is not interference from the steps. Moreover, the noise level does not affect the localization results, which indicates that the proposed method is quite robust to noise.

4. Experimental validation

To further validate the proposed crack localization method, experiments are performed for different crack and step conditions. The basic experiment rig is shown in Fig.7. In order to measure the real SCDS of the rotor, distributed eddy current displacement sensors are arranged along the shaft in the vertical direction as shown in Fig.8. Because of the limitation of experiment condition, only six sensors are used, which will affect the resolution of the localization results that can be improved by using more measurement points or using a more advanced measurement instrument such as a laser vibrometer. In the experiment, the crack is created by wire cutting with 0.2mm width. Though this is not a real fatigue crack and there will be no breathing nonlinearity of the manufactured crack, the 2X frequency component will also be generated by the asymmetry of the crack [38], and since no other obvious asymmetry factors are involved into the experimental rotor, the crack could be exclusively localized by using the 2X SCDS with the proposed method. Therefore, the crack localization method can be validated by this fabricated crack. The two steps are created by creating a ring slot in the shaft. These crack and step configurations for experiment are tabulated in Tab.3. The crack depths in the experiment are 1.57mm, 1.54mm and 3.29mm, which equals to 15.7%, 15.4% and 32.9% of the shaft diameter, respectively. It should be noticed that crack depths less than 25% shaft diameter can be considered as incipient cracks which are hardly observable in simple vibration analysis [39, 40]. The rotating speed of the experimental rotor is 600r/min which is less than one third of the first critical speed to make the 2X component away from the critical rotating frequency, and the sampling frequency is 5000Hz.

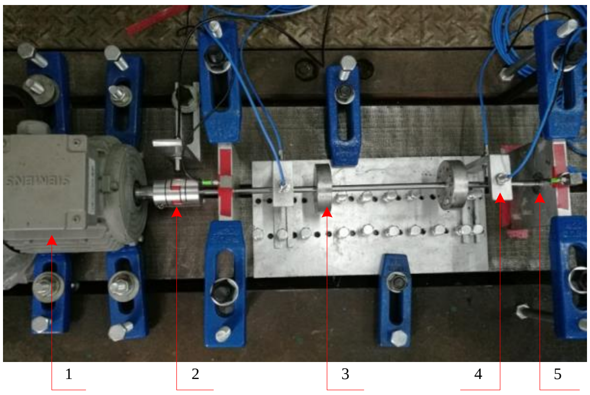

Fig.7 Basic rotor-bearing system experiment rig: (1) Motor; (2) Flexible coupling; (3) Disc; (4) Eddy current displacement sensors; (5) Bearings.

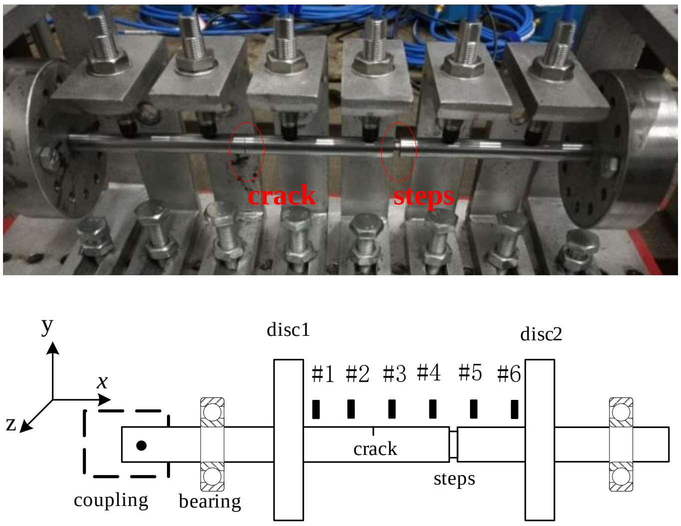

Fig.8 Experimental rotor and its sensor arrangement

Tab.3 Experimental rotor cases for crack localization

Case	Crack location (measurement point)	Crack depth (mm)	Step location (measurement point)
1	3-4	1.57	--
2	2-3	1.54	4-5
3	2-3	3.29	4-5

The responses obtained for case 2 are shown in Fig.9. As can be seen from Fig.9, the responses in all measurement points are polluted by heavy noise, which presents a high demand on the robustness of a crack localization method. Fig.10 shows the localization results for the three cases in Tab.3 based on the proposed method using the 1X and 2X SCDSs. As the nonlinearity induced by the asymmetry of crack mainly appears in 2X component, no higher order SCDSs are considered. In each case of Fig.10, the top graph shows the extracted 1X and 2X SCDSs along with the measurement point, and the bottom graph is the crack localization results corresponding by the proposed method.

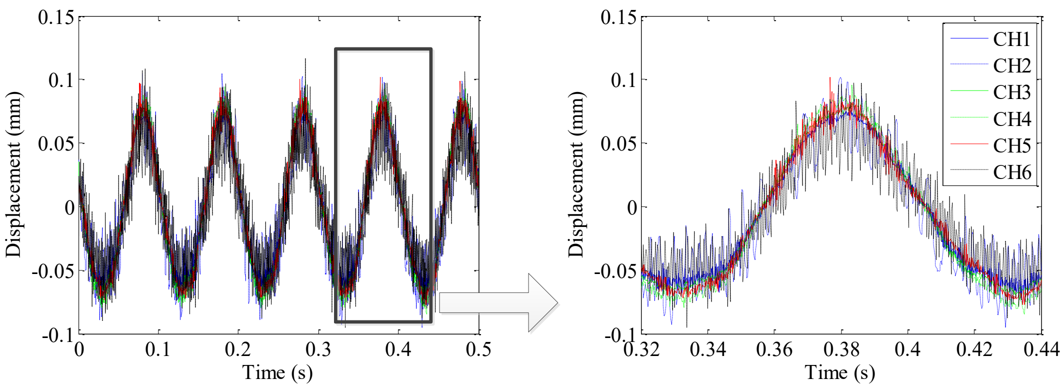

Fig.9 Typical time domain responses of the rotor in case 2

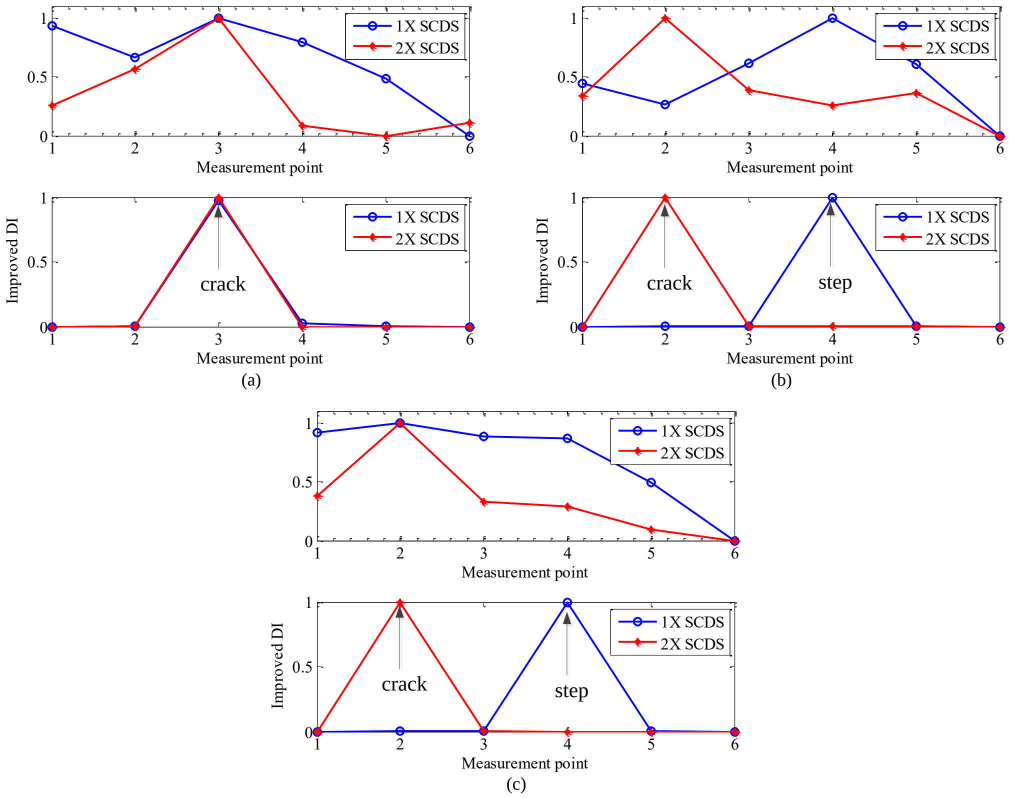

Fig.10 Localization results for experimental rotors: (a) case1; (b) case2; (c) case3.

It can be seen from Fig.10 (a) that both localization results for case 1, where there is no step, based on 1X and 2X SCDSs are accurate. While from Figs.10 (b) corresponding to case 2, where there are steps in the rotor, when 1X SCDS is used for crack localization by the proposed method, a clear peak exists at the location of the step but no obvious peak can be found at the location of the crack. The results show that the interference of the steps cannot be eliminated and misleading localization results are produced. However, when 2X SCDS is utilized for crack localization, only a peak appears at the crack location. Thus the crack localization result is accurate, which means the interference of the steps can be avoided. When increasing the depth of the crack, similar results can be obtained for the stepped rotor in case 3, which can be seen in Fig.10(c). It is well known that the shallower are the cracks, the more difficult will be the crack localization. From the comparison of case 2 with a 1.54mm crack and case 3 with a 3.29mm crack, it manifests that the proposed method performs well under different crack depth and shows high crack depth robustness. Therefore, from the experimental results, it can be concluded that the proposed crack localization method based on Bayesian fusion of multi-scale SCDSs is effective and accurate for stepped rotors, and it is also robust against noise and thus useful in real applications.

5. Discussions

In order to show the advantages of the proposed method, localization results obtained using damage indexes based on SCDSs by gapped smoothing method (GSM) [34] are compared. The basic idea of this published method is to amplify the weak singularities induced by cracks in the SCDS without an intact baseline model (as a reference) by a kind of polynomial curve fitting method called GSM. The squared difference between the gapped polynomial function and the corresponding value of the actual SCDS is constructed as the damage index (DI).

The published GSM method is applied to both numerical and experimental rotors in the current investigation. Fig.11 gives the localization results from simulation 3 with different levels of noise. It shows that when there is no noise the GSM method performs well based on the 2X SCDS. However, with the noise level increases, it becomes increasingly difficult to identify the cracks. Therefore, compared with the proposed method in this work and its localization result in Fig.6, it indicates that the proposed method and the GSM method all performs well when there is no noise or the noise level is low, but the proposed method shows higher robustness against heavy noise and thus would perform better in real applications.

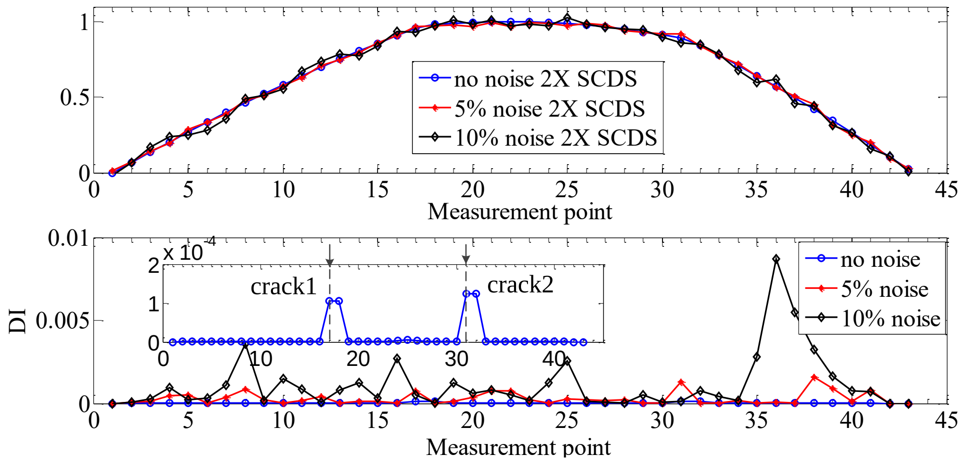

Fig.11 Localization results for the rotor in simulation 3 under different level of noise by GSM method

Further, the GSM method and the proposed method are compared for the crack localization of experimental rotors in Tab.3, and the localization results are shown in Fig.12.

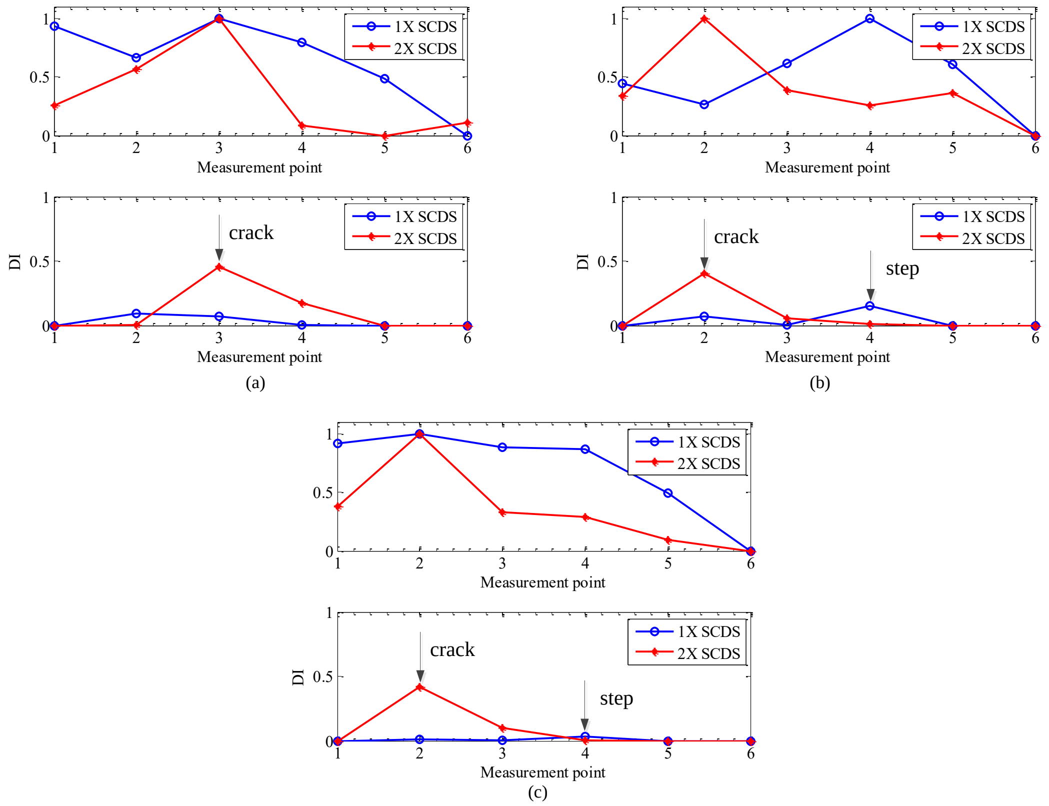

Fig.12 Localization results for experimental rotors by GSM method: (a) case1; (b) case2; (c) case3.

Compared with the localization results in Fig.10 by the proposed method, one can see from Fig.12 that the crack localization method by the GSM method is also accurate when the 2X SCDS is utilized for crack localization. The reason is that the noise in the experimental rotor is not that heavy, so that both the proposed and GSM methods perform well. However, from both the numerical and experimental results, the proposed method shows much better robustness, especially in very noisy cases.

In addition, another property of the proposed crack localization method should be mentioned which is that the proposed method can cope with the interferences from other common nonlinear faults in rotors such as misalignment and initial bow. Actually, this conclusion has been validated by the experiment results, as no strict alignment is performed for the experimental rotors. Thus, coupling misalignment due to installation and shaft bow due to crack manufacturing are already in the experimental rotors. But, the crack localization results are not affected by the coupling faults. The reason is that though the misalignment and initial bow will generate super-harmonic components, they will not affect the smoothness of the deflection of a rotor, thus they will not affect the crack localization results by the proposed method.

6. Conclusions

A novel crack localization method based on Bayesian fusion of multi-scale super-harmonic characteristic deflection shapes (SCDSs) is proposed for stepped rotating rotors by extracting the crack-induced local distortions in the multi-scale SCDSs. In the proposed method, the SCDSs are transformed into multi-scale SCDSs by Gaussian multi-scale analysis, and the multi-scale SCDSs are used to construct a new damage index by Bayesian fusion theory. Numerical simulations for rotors with steps and breathing cracks are carried out, and the numerical results show that the proposed method is effective, accurate and robust, which is also validated by experimental results on a laboratory rotor. Moreover, the proposed method is compared with a recently published crack localization method. The comparison shows the advantage of the proposed method against noise. The proposed method is output-only, and there is no need to know the excitations and build a reference model of an intact baseline model which always difficult to obtain in real rotors, and the common existing steps in rotors will not affect the proposed method. Moreover, some nonlinear factors in rotors such as misalignment and initial bow, though they will generate super-harmonics in the vibration response, they will not affect the smoothness of the deflection of a rotor, thus will not affect the proposed method. Those advantages make the proposed method effective for crack localization in operating rotors even with coupled faults. However, the proposed method is limited for the cracked rotors, thus the existing of cracks needs to be guaranteed by crack detection method before the application of the proposed crack localization method. And higher crack localization accuracy depends on more measurement points, which might be the drawback of the proposed method when applied to real machines, which could be mitigated by advanced testing techniques, such as distributed optical fiber sensing technology.

Abbreviation

BF: Bayesian fusion
CCLP: Crack Closure Line Position
CDS: Characteristic deflection shape
DOFs: Degrees-of-freedom
GMA: Gaussian multi-scale analysis
ODS: Operational deflection shape
PHM: Prognostics and health management
SCDS: Super-harmonic characteristic deflection shapes
SHM: System/structural health monitoring
TEO: Teager energy operator
TODS: Transmissibility of operational deflection shape

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Conflict of interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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